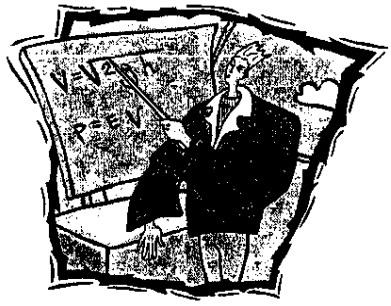
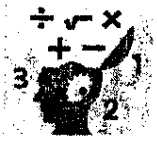
Chapter 6: Radicals





6.1: An Introduction to Square Roots

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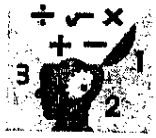


6.1: An Introduction to Square Roots

What are square roots and when are they used?

Square Root of x– the real number that when multiplied by itself, produces x

$\sqrt{\ }$ - radical symbol

$\sqrt{36}$ - the 36 in this example is the radicand

Example: Solve: $\sqrt{36}$

Solution:

$\sqrt{36} = 6$ because $6 \cdot 6 = 36$

List of perfect squares:

1 4 9 16 25 36 49 64 81 100 121 144

Practice Examples:

$\sqrt{4} = 2$ $\sqrt{16} = 4$ $\sqrt{1} = 1$

$\sqrt{25} = 5$ $\sqrt{121} = 11$ $\sqrt{169} = 13$

Estimate Square Roots

Estimate the $\sqrt{45}$

Solution:

$\sqrt{36}$ $\sqrt{45}$ $\sqrt{49}$

6 ≈ 6.7 7

The $\sqrt{45}$ is between the $\sqrt{36}$ which is 6 and $\sqrt{49}$ which is 7.

So the $\sqrt{45}$ is approximately 6.7.

Estimate the following square roots.

$\sqrt{4}$ $\sqrt{6}$ $\sqrt{9}$

2 ≈ 2.5 3

$\sqrt{100}$ $\sqrt{115}$ $\sqrt{121}$

10 ≈ 10.7 11

$\sqrt{81}$ $\sqrt{90}$ $\sqrt{100}$

9 ≈ 9.5 10

$\sqrt{49}$ $\sqrt{55}$ $\sqrt{64}$

7 ≈ 7.4 8

Square Roots of Negatives

1. The square root of a perfect square is a rational number.

$$\sqrt{100} = 10$$

2. The square root of a positive non perfect square is an irrational number.

$$\sqrt{7} \approx 2.645751311....$$

3. The square root of a negative number is a non real number.

$$\sqrt{-4} = \textit{No real solution}$$

$\sqrt{4}$ $\sqrt{-1}$

2 i

Square Roots of Variable Expressions

$\sqrt{9} = 3$ because $\sqrt{9} = \sqrt{3 \cdot 3} = 3$

$\sqrt{9} = \sqrt{3 \cdot 3} = 3$

Since we have pair, the square root of 9 is 3.

The same is true for variables.

$\sqrt{x^2} = x$ because $\sqrt{x^2} = \sqrt{x \cdot x} = x$

$\sqrt{x^2} = \sqrt{x \cdot x} = x$

$\sqrt{x^4} = x^2$ because $\sqrt{x^4} = \sqrt{(x \cdot x) \cdot (x \cdot x)} = x^2$

Find the pattern:

$\sqrt{x^6} = \sqrt{(x\,x)(x\,x)(x\,x)}$ $\boxed{x^3}$

$\sqrt{x^8} = \sqrt{(x\,x)(x\,x)(x\,x)(x\,x)}$ $\boxed{x^4}$

$\sqrt[2]{x^{10}} = x^{\frac{10}{2}} = x^5$

What is the pattern? Varies

Practice Examples:

$\sqrt{x^{20}}$ $\boxed{x^{10}}$

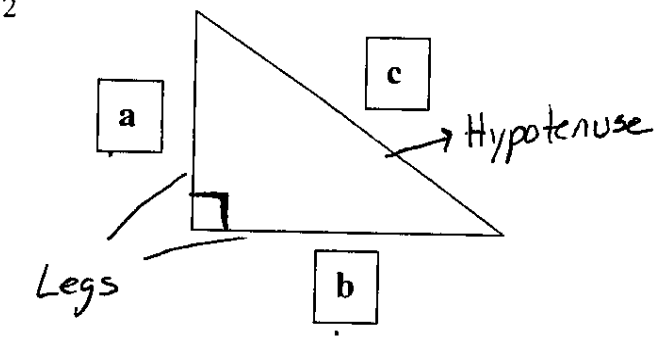
$\sqrt{25x^{14}}$ $\boxed{5x^7}$

$\sqrt{49x^{18}y^6}$ $\boxed{7x^9y^3}$

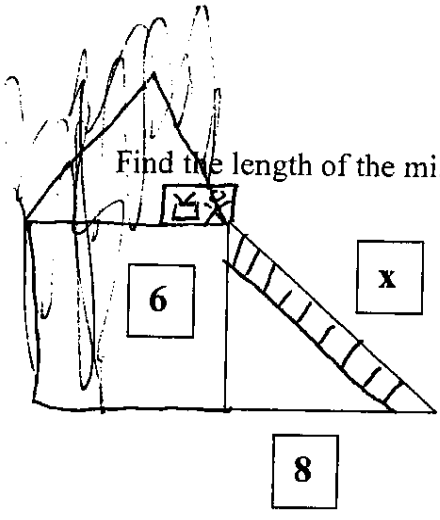
$\sqrt{36x^7}$ $\boxed{6x^3\sqrt{x}}$

Pythagorean Theorem- In a right triangle, the sum of the squares of the lengths of the two shorter sides (legs) is equal to the square of the length of the longest side (hypotenuse).

Pythagorean Theorem: $a^2 + b^2 = c^2$



Find the length of the missing side.



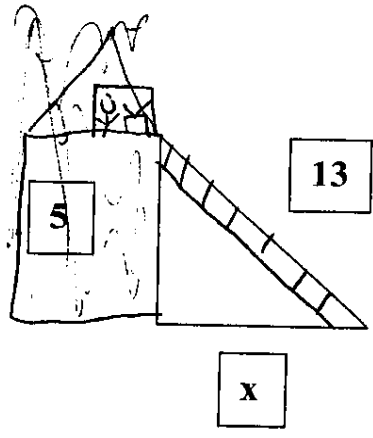
$a^2+b^2=c^2$

$6^2+8^2=c^2$

$36+64=c^2$

$\sqrt{100}=\sqrt{c^2}$

$c=10$ ft



$a^2+b^2=c^2$

$5^2+b^2=13^2$

$25+b^2=169$

$-25 \quad -25$

$b^2=144$

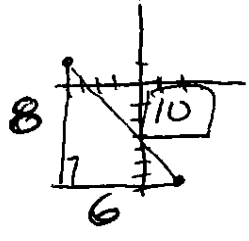
$b=12$

Distance Formula- the distance d between the points with coordinates (x_1, x_2) and (y_1, y_2)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Find the distance between: $(2,-7)$ and $(-4,1)$

$x_1\ y_1 \qquad x_2\ y_2$



$d=\sqrt{(-4-2)^2+(1-(-7))^2}$

$d=\sqrt{(-6)^2+(8)^2}$

$d=\sqrt{36+64}=\sqrt{100}=10$

What are square roots and when are they used?

Varies

6.1: An Introduction to Square Roots Practice Problems

Simplify

1. $\sqrt{9}$ 3 2. $\sqrt{36}$ 6 3. $\sqrt{100}$ 10

4. $\sqrt{144}$ 12 5. $\sqrt{625}$ 25 6. $\sqrt{10{,}000}$ 100

Estimate the following square roots.

7. $\sqrt{3}$ ≈ 1.7

$\sqrt{1}$ $\sqrt{3}$ $\sqrt{4}$
1 2

8. $\sqrt{30}$ ≈ 5.5

$\sqrt{25}$ $\sqrt{30}$ $\sqrt{36}$
5 6

9. $\sqrt{51}$ ≈ 7.1

$\sqrt{49}$ $\sqrt{51}$ $\sqrt{64}$
7 8

10. $\sqrt{92}$ ≈ 9.6

$\sqrt{81}$ $\sqrt{92}$ $\sqrt{100}$
9 10

Simplify:

11. $\sqrt{x^6}$ x^3

12. $\sqrt{x^{30}}$ x^{15}

6.1: An Introduction to Square Roots Practice Problems Continue

Simplify:

13. $\sqrt{4x^2}$ $2x$

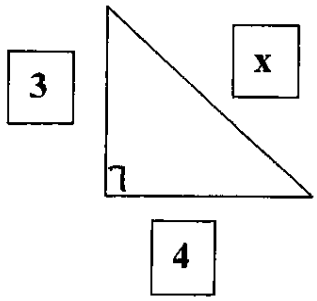
14. $\sqrt{81x^4y^{12}}$ $9x^2y^6$

15. $\sqrt{100x^8y^{18}z^{10}}$ $10x^4y^9z^5$

16. $\sqrt{144x^{50}y^{200}}$ $12x^{25}y^{100}$

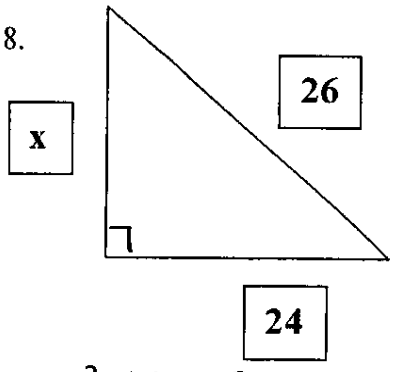
Find the length of the missing side.

17.



$a^2+b^2=c^2$
$3^2+4^2=x^2$
$9+16=x^2$
$\sqrt{x^2}=\sqrt{25}$
$x=\pm 5$ Length is (+)
$x=5$

18.



$a^2+b^2=c^2$
$x^2+(24)^2=(26)^2$
$x^2+576=676$
$-576 \quad -576$
$\sqrt{x^2}=\sqrt{100}$
$x=\pm 10$
$x=10$

19. Find the distance between:
$(-2,9)$ and $(1,5)$ (x_1, y_1 and x_2, y_2)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$d=\sqrt{(1-(-2))^2+(5-9)^2}$

$d=\sqrt{(3)^2+(-4)^2}$

$d=\sqrt{9+16}$

$d=\sqrt{25}=5$

20. Find the distance between:
$(-3,5)$ and $(-6,-2)$ (x_1, y_1 and x_2, y_2)

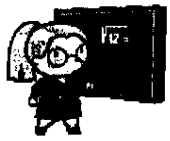
$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$d=\sqrt{(-6-(-3))^2+(-2-5)^2}$

$d=\sqrt{(-3)^2+(-7)^2}$

$d=\sqrt{9+49}$

$d=\sqrt{58}\approx 7.6$



6.2: Simplify Square Roots

How do you simplify square roots?

Simplify Square Roots

Simplify $\sqrt{18}$

Method 1: Factor radicand into prime factors and find pairs

Step 1: Factor radicand into prime factors

$\sqrt{18}$

$\sqrt{3 \cdot 3 \cdot 2}$

Less Thinking
More Work

$\sqrt{18}$

$9 \quad 2$

$3 \cdot 3 \cdot 2$

$\Rightarrow \sqrt{3 \cdot 3} \cdot 2$

$3\sqrt{2}$

Step 2: Circle the pairs

$\sqrt{(3 \cdot 3) \cdot 2}$

Step 3: For every pair one number comes out of the radical

$3\sqrt{2}$

Method 2: Find perfect squares

Step 1: Determine 2 numbers that multiply to 18, where one of the numbers is a perfect square. Then separate the numbers into 2 radicals.

$\sqrt{18}$

$\sqrt{9}\sqrt{2}$

More Thinking
Less Work

$\sqrt{18}$

can $\sqrt{9}$ can't $\sqrt{2}$

Step 2: Take the square root of the perfect square.

$\sqrt{9}\sqrt{2}$

$3\sqrt{2}$

$3\sqrt{2}$

Simplify completely the following square roots

$\sqrt{12}$

$\sqrt{12}$

$\sqrt{2 \cdot 2 \cdot 3}$

$2\sqrt{3}$

$\sqrt{12}$

$\sqrt{4}\sqrt{3}$

$2\sqrt{3}$

$\sqrt{200}$

200

100 2

50 2 2

25 2 2 2

$\sqrt{5 \cdot 5}\sqrt{2 \cdot 2} \cdot 2$

$5 \cdot 2\sqrt{2}$

$10\sqrt{2}$

$\sqrt{100}\sqrt{2}$

$10\sqrt{2}$

$\sqrt{4}\sqrt{50}$

$2\sqrt{50}$

$2\sqrt{25}\sqrt{2}$

$2 \cdot 5\sqrt{2}$

$10\sqrt{2}$

$\sqrt{72}$

$\sqrt{36}\sqrt{2}$

$6\sqrt{2}$

$\sqrt{9}\sqrt{8}$

$3\sqrt{4}\sqrt{2}$

$3 \cdot 2\sqrt{2}$

$6\sqrt{2}$

$\sqrt{3 \cdot 3}\sqrt{2 \cdot 2}\sqrt{2}$

$3 \cdot 2\sqrt{2}$

$6\sqrt{2}$

$5\sqrt{20}$

$5 \quad \sqrt{4}\sqrt{5}$

$5 \cdot 2\sqrt{5}$

$10\sqrt{5}$

$5\sqrt{2 \cdot 2}\sqrt{5}$

$5 \cdot 2\sqrt{5}$

$10\sqrt{5}$

$\sqrt{32}$

$\sqrt{16}\sqrt{2}$

$4\sqrt{2}$

$\sqrt{150}$

$\sqrt{3 \cdot 5 \cdot 2 \cdot 5}$

$5\sqrt{6}$

$\sqrt{25}\sqrt{6}$

$5\sqrt{6}$

Simplify $\sqrt{x^9}$

Method 1: Factor radicand into prime factors and find pairs

Step 1: Factor radicand into prime factors

$\sqrt{x^9}$

$\sqrt{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$

Step 2: Circle the pairs

$\sqrt{(x \cdot x) \cdot (x \cdot x) \cdot (x \cdot x) \cdot (x \cdot x) \cdot x}$

Step 3: For every pair one number comes out of the radical

$x^4\sqrt{x}$

Method 2: Find perfect squares

Step 1: Separate the radicand into perfect squares and non perfect squares.

$\sqrt{x^9}$

$\sqrt{x^8}\sqrt{x}$

Step 2: Take the square root of the perfect squares.

$\sqrt{x^8}\sqrt{x}$

$x^4\sqrt{x}$

Practice Examples:
Simplify

$\sqrt{x^{11}}$ $\sqrt{x^{10}}\sqrt{x}$

$x^5\sqrt{x}$

$\sqrt{x^6 y^{15}}$

can $\sqrt{x^6 y^{14}}$ can't $\sqrt{y}$

$x^3 y^7 \sqrt{y}$

$\sqrt{x^{81}}$

$\sqrt{x^{80}}\sqrt{x}$

$x^{40}\sqrt{x}$

Simplify $\sqrt{20x^5y^2}$

Method 1: Factor radicand into prime factors and find pairs

Step 1: Factor radicand into prime factors

$\sqrt{20x^5y^2}$

$\sqrt{2\cdot 2\cdot 5\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y}$

Step 2: Circle the pairs

$\sqrt{(2\cdot 2)\cdot 5\cdot (x\cdot x)\cdot (x\cdot x)\cdot x\cdot (y\cdot y)}$

Step 3: For every pair one number comes out of the radical

$2x^2y\sqrt{5x}$

Method 2: Find perfect squares

$\sqrt{20x^5y^2}$

Step 1: Separate the radicand into perfect squares and non perfect squares.

can	can't
$\sqrt{4x^4y^2}$	$\sqrt{5x}$

$\sqrt{20x^5y^2}$

$\sqrt{4x^4y^2}\cdot\sqrt{5x}$

$2x^2y\sqrt{5x}$

Step 2: Take the square root of the perfect squares.

$\sqrt{4x^4y^2}\cdot\sqrt{5x}$

$2x^2y\sqrt{5x}$

Practice Examples:
Simplify

$\sqrt{12x^5}$

can: $\sqrt{4x^4}$ can't: $\sqrt{3x}$

$2x^2\sqrt{3x}$

$\sqrt{100y^3}$

can: $\sqrt{100y^2}$ can't: $\sqrt{y}$

$10y\sqrt{y}$

$\sqrt{225yy} = 2 \cdot 5 y \sqrt{y}$

$10y\sqrt{y}$

$5\sqrt{45x^5y^{12}}$

$5\sqrt{9x^4y^{12}}\sqrt{5x}$

$5 \cdot 3x^2y^6\sqrt{5x}$

$15x^2y^6\sqrt{5x}$

$2\sqrt{25x^{10}y}$

$2\sqrt{25x^{10}}\sqrt{y}$ (can / can't)

$2 \cdot 5x^5\sqrt{y}$

$10x^5\sqrt{y}$

$3x^2\sqrt{15x^7y^2}$

$3x^2\sqrt{x^6y^2}\sqrt{15x}$ (can / can't)

$3x^2 x^3 y\sqrt{15x}$

$3x^5y\sqrt{15x}$

$2x^2y\sqrt{50x^{14}y^5z}$

$2x^2y\sqrt{25x^{14}y^4}\sqrt{2yz}$ (can / can't)

$2x^2y \cdot 5x^7y^2\sqrt{2yz}$

$10x^9y^3\sqrt{2yz}$

Quotient Rule to Simplify Square Roots

The square root of the quotient of two numbers is equal to the quotient of their square roots.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Simplify: $\sqrt{\frac{9}{16}}$ $\quad \frac{\sqrt{9}}{\sqrt{16}} = \boxed{\frac{3}{4}}$

Simplify: $\sqrt{\frac{101}{25}}$ $\quad \frac{\sqrt{101}}{\sqrt{25}} = \boxed{\frac{\sqrt{101}}{5}}$

Simplify: $\sqrt{\frac{50a^5b^7}{8ab^{10}}} = \sqrt{\frac{25a^4}{4b^3}} = \frac{\sqrt{25a^4}}{\sqrt{4b^3}} = \frac{5a^2}{2b\sqrt{b}}$

How do you simplify square roots?

Varies

6.2: Simplify Square Roots Practice Problems

Simplify:

1. $\sqrt{28}$ $2\sqrt{7}$

$\sqrt{4}\sqrt{7}$

$2\sqrt{7}$

2. $\sqrt{54}$ $3\sqrt{6}$

$\sqrt{9}\sqrt{6}$

$3\sqrt{6}$

3. $2\sqrt{125}$ $10\sqrt{5}$

$2\sqrt{25}\sqrt{5}$

$2(5)\sqrt{5}$

$10\sqrt{5}$

4. $\sqrt{48}$ $4\sqrt{3}$

$\sqrt{16}\sqrt{3}$

$4\sqrt{3}$

5. $3\sqrt{162}$ $18\sqrt{2}$

$3\sqrt{81}\sqrt{2}$

$3(9)\sqrt{2}$

$18\sqrt{2}$

6. $\sqrt{108}$ $6\sqrt{3}$

$\sqrt{36}\sqrt{3}$

$6\sqrt{3}$

7. $\sqrt{x^3}$ $x\sqrt{x}$

$\sqrt{x^2}\sqrt{x}$

$x\sqrt{x}$

8. $\sqrt{x^9}$ $x^4\sqrt{x}$

$\sqrt{x^8}\sqrt{x}$

$x^4\sqrt{x}$

9. $\sqrt{x^6y^5}$ $x^3y^2\sqrt{y}$

$\sqrt{x^6y^4}\sqrt{y}$

$x^3y^2\sqrt{y}$

10. $x\sqrt{x^7}$ $x^4\sqrt{x}$

$x\sqrt{x^6}\sqrt{x}$

$x\ x^3\sqrt{x}$

$x^4\sqrt{x}$

11. $x^2\sqrt{x^{10}}$ x^7

$x^2\sqrt{x^{10}}$

[redacted]

$x^2\ x^5$

x^7

12. $x^3y\sqrt{x^{11}y^{14}}$ $x^8y^8\sqrt{x}$

$x^3y\sqrt{x^{10}y^{14}}\sqrt{x}$

$x^3y\ x^5y^7\sqrt{x}$

$x^8y^8\sqrt{x}$

6.2: Simplify Square Roots Practice Problems Continue

Simplify:

13. $\sqrt{40x^3}$ $2x\sqrt{10x}$

$\sqrt{4x^2}\ \sqrt{10x}$

$2x\sqrt{10x}$

14. $\sqrt{49y^{11}}$ $7y^5\sqrt{y}$

$\sqrt{49y^{10}}\ \sqrt{y}$

$7y^5\sqrt{y}$

15. $2\sqrt{50x^6y^{21}}$ $10x^3y^{10}\sqrt{2y}$

$2\sqrt{25x^6y^{20}}\sqrt{2y}$

$2(5x^3y^{10})\sqrt{2y}$

$10x^3y^{10}\sqrt{2y}$

16. $3x\sqrt{81x^{25}y^{100}}$ $27x^{13}y^{50}\sqrt{x}$

$3x\sqrt{81x^{24}y^{100}}\sqrt{x}$

$3x(9x^{12}y^{50})\sqrt{x}$

$27x^{13}y^{50}\sqrt{x}$

17. $4xy\sqrt{45x^{16}y^{27}}$ $12x^9y^{14}\sqrt{5y}$

can: $\sqrt{9x^{16}y^{26}}$ can't: $\sqrt{5y}$

$4xy\cdot 3x^8y^{13}\sqrt{5y}$

$12x^9y^{14}\sqrt{5y}$

18. $5x^2y^3\sqrt{72x^3y^8z}$ $30x^3y^7\sqrt{2xz}$

$5x^2y^3\sqrt{36x^2y^8}\sqrt{2xz}$

$5x^2y^3\cdot 6xy^4\sqrt{2xz}$

$30x^3y^7\sqrt{2xz}$

19. $\sqrt{\frac{36}{121}}$ $\frac{6}{11}$

$\frac{\sqrt{36}}{\sqrt{121}} = \frac{6}{11}$

20. $\sqrt{\frac{27a^6b^5}{20a^3b^{15}}}$ $\frac{3a\sqrt{3a}}{2b^5\sqrt{5}}$

$\sqrt{\frac{27a^6b^5}{20a^3b^{15}}}$

$\sqrt{\frac{27\ a^3}{20\ b^{10}}}$

$\frac{\sqrt{27a^3}}{\sqrt{20b^{10}}} = \frac{\sqrt{9a^2}\sqrt{3a}}{\sqrt{4b^{10}}\sqrt{5}} = \frac{3a\sqrt{3a}}{2b^5\sqrt{5}}$

6.3: Adding and Subtracting Radical Expressions

How do you add and subtract radical expressions?

Review Problems
Simplify

Ex. $3x + 4x$

$7x$

Ex. $6x - 3y - 4x + 8y$

$2x + 5y$

Ex. $2x^2 - 3x + 5x^2$

$7x^2 - 3x$

Like Radicals: Square root radicals are called like radicals when they have the same radicand.

Ex. Like Radicals $5\sqrt{3} + 2\sqrt{3}$

$7\sqrt{3}$

Ex. Unlike Radicals $2\sqrt{5} + 7\sqrt{6}$

Combining Like Radicals

Ex. $3\sqrt{2} + 5\sqrt{2}$

$8\sqrt{2}$

Ex. $2\sqrt{3} + 4\sqrt{5} - 6\sqrt{3} + \sqrt{5}$

$-4\sqrt{3} + 5\sqrt{5}$

$5\sqrt{5} - 4\sqrt{3}$

Ex. $2\sqrt{x} + 7\sqrt{x}$

$9\sqrt{x}$

Ex. $7\sqrt{3x} + 2\sqrt{3x}$

$9\sqrt{3x}$

Ex. $7\sqrt{ab} + 2\sqrt{a^2b} - 3\sqrt{ab}$

$4\sqrt{ab} + 2\sqrt{a^2b}$

Ex. $5 + 3\sqrt{x} + 4x$

Simplifying Radicals

Example: $\sqrt{50}+\sqrt{32}$

Solution:

Step 1: Simplify the radicals

$\sqrt{50}+\sqrt{32}$

$\sqrt{25}\cdot\sqrt{2}+\sqrt{16}\cdot\sqrt{2}$

$5\sqrt{2}+4\sqrt{2}$

Step 2: Combine Like Terms

$5\sqrt{2}+4\sqrt{2}$

$9\sqrt{2}$

Practice Examples:

Ex. $\sqrt{45}-\sqrt{27}$

$\sqrt{9}\sqrt{5}-\sqrt{9}\sqrt{3}$

$3\sqrt{5}-3\sqrt{3}$

Ex. $5\sqrt{8}+2\sqrt{18}$

$5\sqrt{4}\sqrt{2}+2\sqrt{9}\sqrt{2}$

$5\cdot 2\sqrt{2}+2\cdot 3\sqrt{2}$

$10\sqrt{2}+6\sqrt{2}=16\sqrt{2}$

Ex. $2\sqrt{24x^2}-x\sqrt{54}$

$2\sqrt{4x^2}\sqrt{6}-x\sqrt{9}\sqrt{6}$ (can, can't)

$2\cdot 2x\sqrt{6}-x\cdot 3\sqrt{6}$

$4x\sqrt{6}-3x\sqrt{6}$

$1x\sqrt{6}$

Ex. $2\sqrt{32x}-6\sqrt{5y}+5\sqrt{200x}+3\sqrt{125y}$

$2\sqrt{16}\sqrt{2x}$ $+5\sqrt{100}\sqrt{2x}$ $+3\sqrt{25}\sqrt{5y}$

$2\cdot 4\sqrt{2x}$ $+5\cdot 10\sqrt{2x}$ $+3\cdot 5\sqrt{5y}$

$8\sqrt{2x}-6\sqrt{5y}+50\sqrt{2x}+15\sqrt{5y}$

$58\sqrt{2x}+9\sqrt{5y}$

How do you add and subtract radical expressions?

Varies

6.3: Adding and Subtracting Radical Expressions Practice Problems

Solve:

1. $4\sqrt{3}+7\sqrt{3}$

$11\sqrt{3}$

2. $5\sqrt{2}+\sqrt{7}-6\sqrt{2}+8\sqrt{7}$

$5\sqrt{2}-6\sqrt{2}+\sqrt{7}+8\sqrt{7}$

$-\sqrt{2}+9\sqrt{7}$

3. $5\sqrt{xy}+4\sqrt{x}-8\sqrt{xy}$

$5\sqrt{xy}-8\sqrt{xy}+4\sqrt{x}$

$-3\sqrt{xy}-4\sqrt{x}$

4. $5x\sqrt{y}+9\sqrt{3x}-8\sqrt{y}+6\sqrt{3x}$

$9\sqrt{3x}+6\sqrt{3x}+5x\sqrt{y}-8\sqrt{y}$

$15\sqrt{3x}+5x\sqrt{y}-8\sqrt{y}$

5. $-4x\sqrt{a^2b^3}+2\sqrt{a^2b^3}-6x\sqrt{a^2b^3}$

$-4x\sqrt{a^2b^2}\sqrt{b}+2\sqrt{a^2b^2}\sqrt{b}-6x\sqrt{a^2b^2}\sqrt{b}$

$-4xab\sqrt{b}+2ab\sqrt{b}-6xab\sqrt{b}$

$-4abx\sqrt{b}-6abx\sqrt{b}+2ab\sqrt{b}$

$-10abx\sqrt{b}+2ab\sqrt{b}$ or

$-10x\sqrt{a^2b^3}+2\sqrt{a^2b^3}$

6. $\sqrt{12}-\sqrt{27}$

$\sqrt{4}\sqrt{3}-\sqrt{9}\sqrt{3}$

$2\sqrt{3}-3\sqrt{3}$

$-\sqrt{3}$

7. $\sqrt{32}+\sqrt{18}$

$\sqrt{16}\sqrt{2}+\sqrt{9}\sqrt{2}$

$4\sqrt{2}+3\sqrt{2}$

$7\sqrt{2}$

6.3: Add and Subtract Radical Expressions Practice Continue

8. $\sqrt{27}-\sqrt{48}+\sqrt{75}$

$\sqrt{9}\sqrt{3}-\sqrt{16}\sqrt{3}+\sqrt{25}\sqrt{3}$

$3\sqrt{3}-4\sqrt{3}+5\sqrt{3}$

$-\sqrt{3}+5\sqrt{3}$

$4\sqrt{3}$

9. $3\sqrt{12}+2\sqrt{48}$

$3\sqrt{4}\sqrt{3}+2\sqrt{16}\sqrt{3}$

$3\cdot 2\sqrt{3}+2\cdot 4\sqrt{3}$

$6\sqrt{3}+8\sqrt{3}$

$14\sqrt{3}$

10. $\sqrt{108}+3\sqrt{75}-2\sqrt{50}$

$\sqrt{36}\sqrt{3}+3\sqrt{25}\sqrt{3}-2\sqrt{25}\sqrt{2}$

$6\sqrt{3}+3\cdot 5\sqrt{3}-2\cdot 5\sqrt{2}$

$6\sqrt{3}+15\sqrt{3}-10\sqrt{2}$

$21\sqrt{3}-10\sqrt{2}$

11. $3\sqrt{8}+\sqrt{48}-5\sqrt{27}-2\sqrt{32}$

$3\sqrt{4}\sqrt{2}+\sqrt{16}\sqrt{3}-5\sqrt{9}\sqrt{3}-2\sqrt{16}\sqrt{2}$

$3\cdot 2\sqrt{2}+4\sqrt{3}-5(3)\sqrt{3}-2(4)\sqrt{2}$

$6\sqrt{2}+4\sqrt{3}-15\sqrt{3}-8\sqrt{2}$

$6\sqrt{2}-8\sqrt{2}+4\sqrt{3}-15\sqrt{3}$

$-2\sqrt{2}-11\sqrt{3}$

12. $5\sqrt{54x^2}-2\sqrt{24x^2}$

$5\sqrt{9x^2}\sqrt{6}-2\sqrt{4x^2}\sqrt{6}$

$5\cdot 3x\sqrt{6}-2\cdot 2x\sqrt{6}$

$15x\sqrt{6}-4x\sqrt{6}$

$11x\sqrt{6}$

13. $2a\sqrt{48ab^2}-b\sqrt{27a^3}+3\sqrt{75a^3b^2}$

$2a\sqrt{16b^2}\sqrt{3a}-b\sqrt{9a^2}\sqrt{3a}+3\sqrt{25a^2b^2}\sqrt{3a}$

$2a\cdot 4b\sqrt{3a}-b\cdot 3a\sqrt{3a}+3\cdot 5ab\sqrt{3a}$

$8ab\sqrt{3a}-3ab\sqrt{3a}+15ab\sqrt{3a}$

$5ab\sqrt{3a}+15ab\sqrt{3a}$

$20ab\sqrt{3a}$

6.4: Multiplying and Dividing Radical Expressions

How do you multiply and divide radical expressions?

Example: $4\sqrt{6} \cdot 5\sqrt{3}$

Solution:

Step 1: To multiply radical expressions, multiply coefficients, multiply radicands

$4\sqrt{6} \cdot 5\sqrt{3}$

$4 \cdot 5\sqrt{6 \cdot 3}$

$20\sqrt{18}$

Step 2: If needed, then simplify radicand

$20\sqrt{18}$

$20\sqrt{9}\sqrt{2}$

$20 \cdot 3\sqrt{2}$

$60\sqrt{2}$

Practice Examples:

Ex. $\sqrt{5} \cdot 3\sqrt{2}$

$3\sqrt{10}$

Ex. $\sqrt{6x} \cdot \sqrt{8x}$

$\sqrt{48x^2}$

$\sqrt{16x^2}\sqrt{3}$

$4x\sqrt{3}$

Ex. $2\sqrt{5x^3} \cdot 3\sqrt{4x^6}$

$6\sqrt{20x^9}$

$6\sqrt{4x^8}\sqrt{5x}$

$6 \cdot 2x^4\sqrt{5x}$

$12x^4\sqrt{5x}$

Ex. $4\sqrt{2x} \cdot 3\sqrt{14y}$

$12\sqrt{28xy}$

$12\sqrt{4}\sqrt{7xy}$

$12 \cdot 2$

$24\sqrt{7xy}$

Square of a square root: $\left(\sqrt{x}\right)^2 = x$, for any positive real number x

Practice Examples:

Ex. $\left(\sqrt{5}\right)^2$

5

Ex. $\left(\sqrt{x-3}\right)^2$

$x-3$

Ex. $\left(3\sqrt{2}\right)^2$

$9 \cdot 2$

$\boxed{18}$

Multiplying Radicals Expressions

Example: $3\sqrt{2x}\left(5\sqrt{4x^3} - \sqrt{3x}\right)$

Solution:

Step 1: Use distributive property

$3\sqrt{2x}\left(5\sqrt{4x^3} - \sqrt{3x}\right)$

$3\sqrt{2x}\left(5\sqrt{4x^3}\right) + 3\sqrt{2x}\left(-\sqrt{3x}\right)$

Step 2: Multiply the radicals

$3\sqrt{2x}\left(5\sqrt{4x^3}\right) + 3\sqrt{2x}\left(-\sqrt{3x^4}\right)$

$15\sqrt{8x^4} - 3\sqrt{6x^5}$

Step 2: Simplify the radicals

$15\sqrt{8x^4} - 3\sqrt{6x^5}$

$15\sqrt{4x^4} \cdot \sqrt{x} - 3\sqrt{x^4}\sqrt{6x}$

$15 \cdot 2x^2\sqrt{x} - 3x^2\sqrt{6x}$

$30x^2\sqrt{x} - 3x^2\sqrt{6x}$

$2\sqrt{3}\left(3\sqrt{6} - 4\sqrt{3}\right)$

$6\sqrt{18} - 8\sqrt{9}$

$6\sqrt{9}\sqrt{2} - 8 \cdot 3$

$6 \cdot 3\sqrt{2} - 24$

$\boxed{18\sqrt{2} - 24}$

Example: $(3\sqrt{2}-4)(\sqrt{2}+3)$

Solution:

Step 1: Use distributive property or FOIL

$(3\sqrt{2}-4)(\sqrt{2}+3)$

$3\sqrt{2}(\sqrt{2})+3\sqrt{2}(3)-4(\sqrt{2})-4(3)$

Step 2: Multiply the radicals

$3\sqrt{2}(\sqrt{2})+3\sqrt{2}(3)-4(\sqrt{2})-4(3)$

$3\sqrt{4}+9\sqrt{2}-4\sqrt{2}-12$

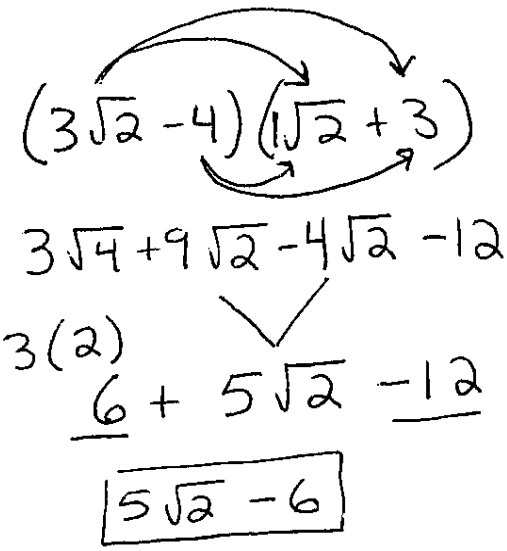
Step 2: Simplify the radicals

$3\sqrt{4}+9\sqrt{2}-4\sqrt{2}-12$

$3(2)+9\sqrt{2}-4\sqrt{2}-12$

$6+5\sqrt{2}-12$

$-6+5\sqrt{2}$



Practice Examples:

Ex. $4\sqrt{5}(3\sqrt{5}-6)$

$12\sqrt{25}-24\sqrt{5}$

$12\cdot 5-24\sqrt{5}$

$60-24\sqrt{5}$

Ex. $2\sqrt{3x}(\sqrt{4x}-3\sqrt{x^2})$

$2\sqrt{12x^2}-6\sqrt{3x^3}$

$2\sqrt{4x^2}\sqrt{3}-6\sqrt{x^2}\sqrt{3x}$

$2\cdot 2x\sqrt{3}$

$4x\sqrt{3}-6x\sqrt{3x}$

Ex. $1\sqrt{2}(7\sqrt{9}-\sqrt{8})$

$7\sqrt{18}-1\sqrt{16}$

$7\sqrt{9}\sqrt{2}-4$

$7\cdot 3\sqrt{2}$

$21\sqrt{2}-4$

$-4+21\sqrt{2}$

Practice Examples:

Ex. $(2\sqrt{3}+5)(\sqrt{5}+2)$

$2\sqrt{15}+4\sqrt{3}+5\sqrt{5}+10$

$\boxed{2\sqrt{15}+4\sqrt{3}+5\sqrt{5}+10}$

Ex. $(3\sqrt{6}-2)(2\sqrt{6}+5)$

$6\sqrt{36}+15\sqrt{6}-4\sqrt{6}-10$

$6\cdot 6+15\sqrt{6}-4\sqrt{6}-10$

$36+11\sqrt{6}-10$

$\boxed{26+11\sqrt{6}}$

Ex. $(\sqrt{7}-3)^2$

$(\sqrt{7}-3)(\sqrt{7}-3)$

$\sqrt{49}-3\sqrt{7}-3\sqrt{7}+9$

$7-6\sqrt{7}+9$

$\boxed{16-6\sqrt{7}}$

Ex. $(\sqrt{3x}-2)(4\sqrt{3x}-6)$

$4\sqrt{9x^2}-6\sqrt{3x}-8\sqrt{3x}+12$

$4\cdot 3x-14\sqrt{3x}+12$

$\boxed{12x-14\sqrt{3x}+12}$

Dividing Radicals Expressions

Practice Examples:

Ex. $\frac{\sqrt{40}}{\sqrt{5}}$

$\sqrt{\frac{40}{5}}$

$\sqrt{8}$

$\sqrt{4}\ \sqrt{2}$

$2\sqrt{2}$

Ex. $\frac{\sqrt{84}}{\sqrt{3}}$

$\sqrt{\frac{84}{3}}$

$\sqrt{28}$

$\sqrt{4}\ \sqrt{7}$

$2\sqrt{7}$

Ex. $\frac{\sqrt{32x^6}}{\sqrt{2x^2}}$

$\sqrt{\frac{32x^6}{2x^2}}$

$\sqrt{16x^4}$

$4x^2$

Ex. $\frac{\sqrt{200x^{11}}}{\sqrt{4x}}$

$\sqrt{\frac{200x^{11}}{4x}}$

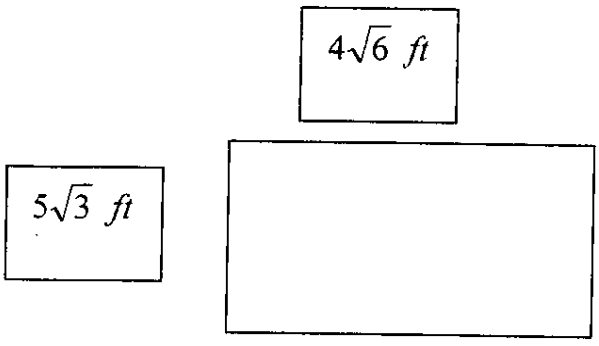
$\sqrt{50x^{10}}$

$\sqrt{25x^{10}}\ \sqrt{2}$

$5x^5\sqrt{2}$

Applications

Find the area of the rectangle.



$(5\sqrt{3})(4\sqrt{6})$

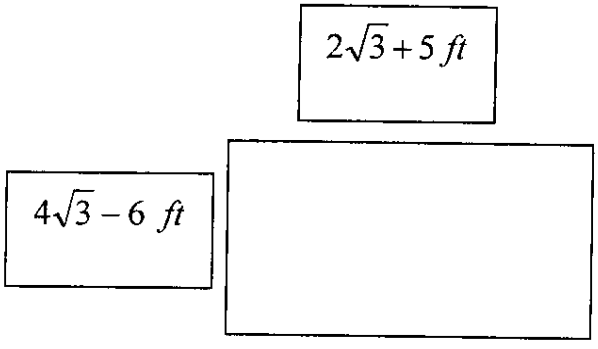
$20\sqrt{18}$

$20\sqrt{9}\sqrt{2}$

$20(3)\sqrt{2}$

$60\sqrt{2}$ ft^2

Find the area of the rectangle.



$(2\sqrt{3}+5)(4\sqrt{3}-6)$

$6\sqrt{9} - 12\sqrt{3} + 20\sqrt{3} - 30$

$6(3) + 8\sqrt{3} - 30$

$18 + 8\sqrt{3} - 30$

$8\sqrt{3} - 12$ ft^2

How do you multiply and divide radical expressions?

Varies

6.4: Multiplying and Dividing Radical Expressions Practice Problems

Simplify

1. $\sqrt{3}(4\sqrt{2}-5)$

$4\sqrt{6}-5\sqrt{3}$

2. $3\sqrt{5}(4\sqrt{2}-\sqrt{6})$

$12\sqrt{10}-3\sqrt{30}$

3. $5\sqrt{2}(2\sqrt{8}-7)$

$10\sqrt{16}-35\sqrt{2}$

$10(4)-35\sqrt{2}$

$40-35\sqrt{2}$

4. $3\sqrt{10}(4\sqrt{2}-6\sqrt{5})$

$12\sqrt{20}-18\sqrt{50}$

$12\sqrt{4}\sqrt{5}-18\sqrt{25}\sqrt{2}$

$12\cdot 2\sqrt{5}-18(5)\sqrt{2}$

$24\sqrt{5}-90\sqrt{2}$

5. $2\sqrt{2x}(\sqrt{14x}-6\sqrt{x^5})$

$2\sqrt{28x^2}-12\sqrt{2x^6}$

$2\sqrt{4x^2}\sqrt{7}-12\sqrt{x^6}\sqrt{2}$

$2(2x)\sqrt{7}-12x^3\sqrt{2}$

$4x\sqrt{7}-12x^3\sqrt{2}$

6. $x\sqrt{3}(x\sqrt{8x^2y}-5xy\sqrt{3x^3y^2})$

$x^2\sqrt{24x^2y}-5x^2y\sqrt{9x^3y^2}$

$x^2\sqrt{4x^2}\sqrt{6y}-5x^2y\sqrt{9x^2y^2}\sqrt{x}$

$x^2(2x)\sqrt{6y}-5x^2y(3xy)\sqrt{x}$

$2x^3\sqrt{6y}-15x^3y^2\sqrt{x}$

7. $(2\sqrt{3}+4)(3\sqrt{2}+5)$

$6\sqrt{6}+10\sqrt{3}+12\sqrt{2}+20$

8. $(5\sqrt{2}+3)(3\sqrt{2}-4)$

$15\sqrt{4}-20\sqrt{2}+9\sqrt{2}-12$

$15(2)-11\sqrt{2}-12$

$30-11\sqrt{2}-12$

$18-11\sqrt{2}$

6.4: Multiplying and Dividing Radical Expressions Problems Continue

9. $(\sqrt{3}+5)^2$

$(\sqrt{3}+5)(\sqrt{3}+5)$

$\sqrt{9}+5\sqrt{3}+5\sqrt{3}+25$

$3+10\sqrt{3}+25$

$28+10\sqrt{3}$

10. $(2\sqrt{6}-7)^2$

$(2\sqrt{6}-7)(2\sqrt{6}-7)$

$4\sqrt{36}-14\sqrt{6}-14\sqrt{6}+49$

$4(6)$

$24-28\sqrt{6}+49$

$73-28\sqrt{6}$

11. $(4\sqrt{3x}+3x)(3\sqrt{15x}-\sqrt{3x})$

$12\sqrt{45x^2}-4\sqrt{9x^2}+9x\sqrt{15x}-3x\sqrt{3x}$

$12\sqrt{9x^2}\sqrt{5}-4(3x)$

$12\cdot 3x\sqrt{5}$

$36x\sqrt{5}-12x+9x\sqrt{15x}-3x\sqrt{3x}$

12. $(5x\sqrt{6xy}-\sqrt{2y})(\sqrt{3x^3}-2x^2\sqrt{2xy})$

$5x\sqrt{18x^4y}-10x^3\sqrt{12x^2y^2}-\sqrt{6x^3y}+2x^2\sqrt{4x^2y^2}$

$5x\sqrt{9x^4}\sqrt{2y}-10x^3\sqrt{4x^2y^2}\sqrt{3}-\sqrt{x^2}\sqrt{6xy}+2x^2\sqrt{4y^2}\sqrt{x}$

$5x(3x^2)\sqrt{2y}-10x^3(2xy)\sqrt{3}-x\sqrt{6xy}+2x^2(2y)\sqrt{x}$

$15x^3\sqrt{2y}-20x^4y\sqrt{3}-x\sqrt{6xy}+4x^2y\sqrt{x}$

13. $\dfrac{\sqrt{54}}{\sqrt{3}}=\sqrt{\dfrac{54}{3}}=\sqrt{18}$

$\sqrt{9}\sqrt{2}$

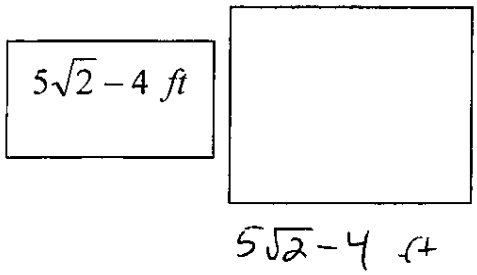
$3\sqrt{2}$

14. $\dfrac{\sqrt{90x^{10}}}{\sqrt{2x^3}}=\sqrt{45x^7}$

$\sqrt{9x^6}\sqrt{5x}$

$3x^3\sqrt{5x}$

15. Find the area of square.



$(5\sqrt{2}-4)(5\sqrt{2}-4)$

$25\sqrt{4}-20\sqrt{2}-20\sqrt{2}+16$

$25(2)$

$50-40\sqrt{2}+16$

$66-40\sqrt{2}$ ft^2

6.5: Solving Radical Equations

How do you solve radical expressions?

Example: Solve for x: $3\sqrt{x+2}+5=14$

Solution:

Step 1: Isolate the radical

$3\sqrt{x+2}+5=14$

$\quad -5 \quad -5$

$3\sqrt{x+2}=9$

$\div 3 \qquad \div 3$

$\sqrt{x+2}=3$

Step 2: Remove the radical by squaring both sides

$\sqrt{x+2}=3$

$\left(\sqrt{x+2}\right)^2=(3)^2$

$x+2=9$

Step 3: Solve for x

$x+2=9$

$\quad -2 \quad -2$

$x=7$

Step 4: Check

$3\sqrt{x+2}+5=14$

$3\sqrt{7+2}+5=14$

$3\sqrt{9}+5=14$

$3\cdot 3+5=14$

$9+5=14$

$14=14$

Practice Examples:

Example: Solve for x: $\sqrt{x-5}-3=4$

$\sqrt{x-5} - 3 = 4$

$+3 \quad +3$

$\sqrt{x-5} = 7$

$(\sqrt{x-5})^2 = (7)^2$

$x-5 = 49$

$+5 \quad +5$

$\boxed{x = 54}$

✓ $\sqrt{54-5} - 3 = 4$

$\sqrt{49} - 3 = 4$

$7 - 3 = 4$

$4 = 4$

Example: Solve for x: $\sqrt{x-2}+5=2$

$\sqrt{x-2} + 5 = 2$

$-5 \quad -5$

Notice this equation →

$\sqrt{x-2} = -3$

$(\sqrt{x-2})^2 = (-3)^2$

$x-2 = 9$

$+2 \quad +2$

$\boxed{x = 11}$

check

✓ $\sqrt{11-2} + 5 = 2$

$\sqrt{9} + 5 = 2$

$3 + 5 = 2$

$8 \neq 2$

Doesn't check so

$\boxed{\text{No Solution}}$

Practice Examples:

Example: Solve for x: $\sqrt{x+4} - x = -2$

$+x \quad +x$

$\sqrt{x+4} = x-2$

$(\sqrt{x+4})^2 = (x-2)^2$

$(x-2)(x-2)$

$x+4 = x^2-2x-2x+4$

$x+4 = x^2-4x+4$

$-x \qquad -x$

$4 = x^2-5x+4$

$-4 \qquad -4$

$x^2-5x=0$

$x^2-5x=0$

$x(x-5)=0$

$x=0$ $\quad x-5=0$

$+5 \quad +5$

$x=5$

Check

$\sqrt{0+4}-0=-2$	$\sqrt{5+4}-5=-2$
$\sqrt{4}-0=-2$	$\sqrt{9}-5=-2$
$2-0=-2$	$3-5=-2$
$2 \neq -2$	$-2=-2$
Doesn't Check	checks

Solution $x=5$

Example: Solve for x: $\sqrt{x-4} = 2\sqrt{x-16}$

$\sqrt{x-4} = 2\sqrt{x-16}$

$(\sqrt{x-4})^2 = (2\sqrt{x-16})^2$

$x-4 = 4(x-16)$

$x-4 = 4x-64$

$-x \qquad -x$

$-4 = 3x-64$

$+64 \qquad +64$

$\frac{60}{3} = \frac{3x}{3}$

$x=20$

✓ Check

$\sqrt{20-4} = 2\sqrt{20-16}$

$\sqrt{16} = 2\sqrt{4}$

$4 = 2(2)$

$4=4$

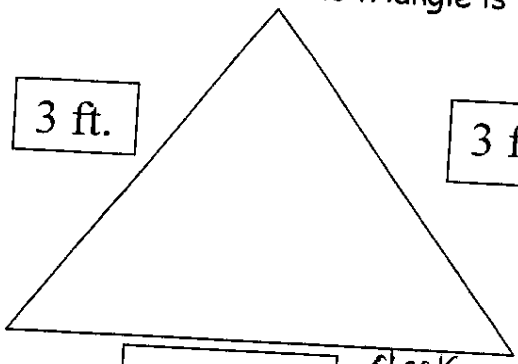
ok

Solution

$x=20$

Applications

Solve for x, and then find the missing side.
The perimeter of the triangle is 13 feet.



$3+3+\sqrt{x+5}=13$

$6+\sqrt{x+5}=13$

$-6 \quad -6$

$\sqrt{x+5}=7$

$(\sqrt{x+5})^2=(7)^2$

$x+5=49$

$-5 \quad -5$

$x=44$

Missing Side

$\sqrt{44+5}$

$\sqrt{49}$

7

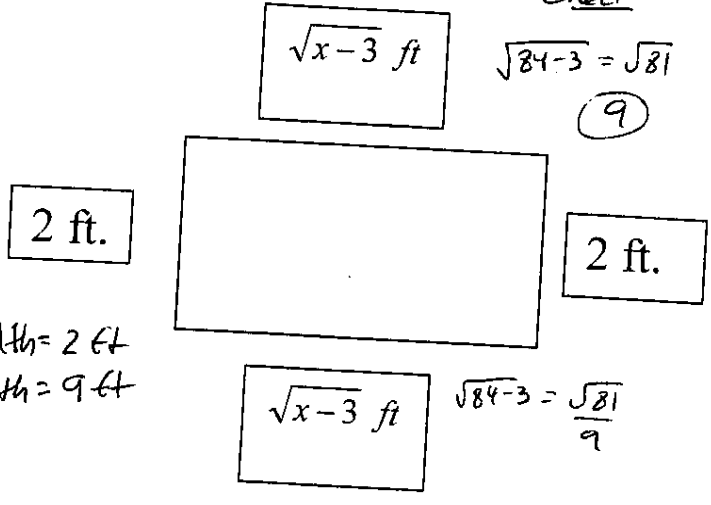
Check

$\sqrt{44+5}$

$\sqrt{49}=7$

$3+3+7=13$

Solve for x, and then find the width and length.
The perimeter of the rectangle is 22 feet.



Check

$\sqrt{84-3}=\sqrt{81}$

9

$\sqrt{84-3}=\sqrt{81}$

9

width = 2 ft
length = 9 ft

$2+\sqrt{x-3}+2+\sqrt{x-3}=22$

$2\sqrt{x-3}+4=22$

$-4 \quad -4$

$\frac{2\sqrt{x-3}}{2}=\frac{18}{2}$

$\sqrt{x-3}=9$

$(\sqrt{x-3})^2=(9)^2$

$x-3=81$

$+3 \quad +3$

$x=84$

Solutions
Width = 2
Length = $\sqrt{84-3}=\sqrt{81}$
9

$2+9+2+9=22$

$22=22$

How do you solve radical expressions?

Varies

6.5: Solving Radical Equations Practice Problems

Solve:

1. $\sqrt{x} = 9$

$(\sqrt{x})^2 = (9)^2$

$\boxed{x = 81}$

✓ $\sqrt{81} = 9$

$9 = 9$

2. $\sqrt{x} = -2$

$(\sqrt{x})^2 = (-2)^2$

$x = 4$

✓ $\sqrt{4} = -2$

$2 \neq -2$

$\boxed{\text{No Solution}}$

3. $\sqrt{2x} = 4$

$(\sqrt{2x})^2 = (4)^2$

$\frac{2x}{2} = \frac{16}{2}$

$\boxed{x = 8}$

✓ $\sqrt{2(8)} = 4$

$\sqrt{16} = 4$

$4 = 4$

4. $\sqrt{x} + 3 = 7$

$-3 \quad -3$

$\sqrt{x} = 4$

$(\sqrt{x}) = 4^2$

$\boxed{x = 16}$

✓ $\sqrt{16} + 3 = 7$

$4 + 3 = 7$

$7 = 7$

5. $\sqrt{x+2} = 3$

$(\sqrt{x+2})^2 = (3)^2$

$x + 2 = 9$

$-2 \quad -2$

$\boxed{x = 7}$

✓ $\sqrt{7+2} = 3$

$\sqrt{9} = 3$

$3 = 3$

6. $\sqrt{x-7} = 10$

$(\sqrt{x-7})^2 = (10)^2$

$x - 7 = 100$

$+7 \quad +7$

$\boxed{x = 107}$

✓ $\sqrt{107-7} = 10$

$\sqrt{100} = 10$

$10 = 10$

7. $\sqrt{2x-4} = 6$

$(\sqrt{2x-4})^2 = (6)^2$

$2x - 4 = 36$

$+4 \quad +4$

$\frac{2x}{2} = \frac{40}{2}$

$\boxed{x = 20}$

✓ $\sqrt{2(20)-4} = 6$

$\sqrt{40-4} = 6$

$\sqrt{36} = 6$

$6 = 6$

8. $\sqrt{x+1} - 6 = 3$

$+6 \quad +6$

$\sqrt{x+1} = 9$

$(\sqrt{x+1})^2 = (9)^2$

$x + 1 = 81$

$-1 \quad -1$

$\boxed{x = 80}$

✓ $\sqrt{80+1} - 6 = 3$

$\sqrt{81} - 6 = 3$

$9 - 6 = 3$

$3 = 3$

9. $\sqrt{3x-14} + 3 = 7$

$-3 \quad -3$

$\sqrt{3x-14} = 4$

$(\sqrt{3x-14})^2 = (4)^2$

$3x - 14 = 16$

$+14 \quad +14$

$\frac{3x}{3} = \frac{30}{3}$

$\boxed{x = 10}$

✓ $\sqrt{3(10)-14} + 3 = 7$

$\sqrt{16} + 3 = 7$

$4 + 3 = 7$

$7 = 7$

6.5: Solving Radical Equations Practice Problems Continue

Solve:

10. $\sqrt{x^2+2x-6} = x$

$\left(\sqrt{x^2+2x-6}\right)^2 = (x)^2$

$x^2+2x-6 = x^2$

$-x^2 \quad -x^2$

$2x-6=0$

$+6 \quad +6$

$\frac{2x}{2} = \frac{6}{2}$

$x=3$

Check

$\sqrt{(3)^2+2(3)-6} = 3$

$\sqrt{9} = 3$

$3 = 3$

11. $\sqrt{x+4} = x-2$

$\left(\sqrt{x+4}\right)^2 = (x-2)^2$

$x+4 = x^2-4x+4$

$-x \quad -4 \qquad -x \quad -4$

$x^2-5x+0=0$

[redacted] Solutions

$x^2-5x=0$

$x(x-5)=0$

$x=0$

$x-5=0$

$+5 \quad +5$

$x=5$

Check

$\sqrt{0+4} = 0-2$

$\sqrt{4} = -2$

$2 \neq -2$

Not a solution

$\sqrt{5+4} = 5-2$

$\sqrt{9} = 3$

$3 = 3$

Solution

Solution

$x=5$

12. $\sqrt{5x+6} = \sqrt{7x-6}$

$\left(\sqrt{5x+6}\right)^2 = \left(\sqrt{7x-6}\right)^2$

$5x+6 = 7x-6$

$-5x \quad -5x$

$6 = 2x-6$

$+6 \quad +6$

$\frac{12}{2} = \frac{2x}{2}$

$x=6$

Check

$\sqrt{5(6)+6} = \sqrt{7(6)-6}$

$\sqrt{36} = \sqrt{36}$

$6 = 6$

13. $2\sqrt{3x+4} = \sqrt{5x+9}$

$\left(2\sqrt{3x+4}\right)^2 = \left(\sqrt{5x+9}\right)^2$

$4(3x+4) = 5x+9$

$12x+16 = 5x+9$

$-5x \quad -5x$

$7x+16 = 9$

$-16 \quad -16$

$\frac{7x}{7} = \frac{-7}{7}$

$x=-1$

Check

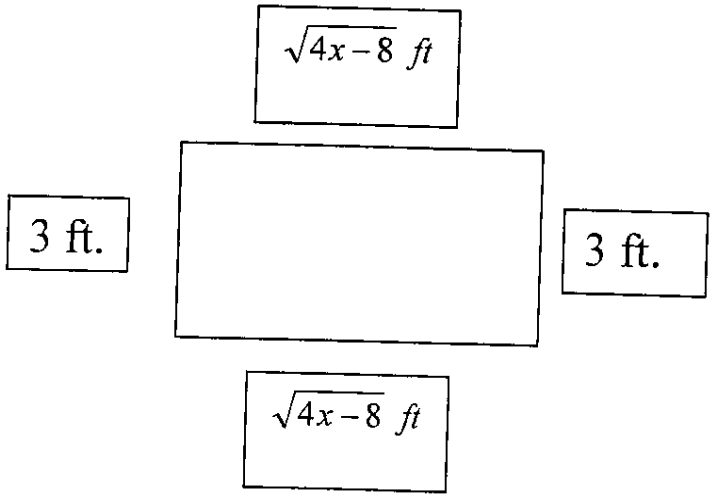
$2\sqrt{3(-1)+4} = \sqrt{5(-1)+9}$

$2\sqrt{1} = \sqrt{4}$

$2(1) = 2$

$2 = 2$

14. Solve for x, and then find the width and length.
The perimeter of the rectangle is 18 feet.



$3+\sqrt{4x-8}+3+\sqrt{4x-8} = 18$

$2\sqrt{4x-8}+6 = 18$

$-6 \quad -6$

$\frac{2\sqrt{4x-8}}{2} = \frac{12}{2}$

$\sqrt{4x-8} = 6$

$\left(\sqrt{4x-8}\right)^2 = (6)^2$

$4x-8 = 36$

$+8 \quad +8$

$\frac{4x}{4} = \frac{44}{4}$

$x=11$

Width = 3 ft

Length = 6 ft

$\sqrt{4(11)-8} = \sqrt{36} = 6$

$\sqrt{4(11)-8} = 6$

3 [ ] 3

$\sqrt{4(11)-8} = 6$

6.6: Higher Order Roots

What are and how do you solve higher order roots?

Higher Order Roots

Square roots are not the only roots. There are many different types of roots.

Here are some examples:

$\sqrt[3]{125} = \sqrt[3]{5 \cdot 5 \cdot 5} = 5$

$\sqrt[4]{81} = \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3} = 3$

$\sqrt[5]{32} = \sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2$

Simplify the following roots:

Practice examples:

Ex. $\sqrt[3]{27}$

$9 \cdot 3$

$3 \cdot 3 \cdot 3$

3

Ex. $\sqrt[4]{16}$

$4 \cdot 4$

$2 \cdot 2 \cdot 2 \cdot 2$

2

ex. $\sqrt[3]{64}$

$8 \cdot 8$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$2 \cdot 2 = 4$

ex. $\sqrt[3]{40}$

$\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5}$

$2\sqrt[3]{5}$

Ex. $\sqrt[3]{x^6}$

$\sqrt[3]{x \cdot x \cdot x \cdot x \cdot x \cdot x}$

x^2

Ex. $\sqrt[5]{x^{15} y^{25}}$

$x^3 y^5$

Practice examples:
Simplify

Ex. $\sqrt[3]{40}$

$\sqrt[3]{2\cdot2\cdot2\cdot5}$ or $\sqrt[3]{8}\ \sqrt[3]{5}$

$2\sqrt[3]{5}$ $\quad 2\sqrt[3]{5}$

Ex. $\sqrt[4]{80}$

$\sqrt[4]{2\cdot2\cdot2\cdot2\cdot5}$ or $\sqrt[4]{16}\ \sqrt[4]{5}$

$2\sqrt[4]{5}$ $\quad 2\sqrt[4]{5}$

Ex. $\sqrt[3]{+128x^6y^5}$

$\sqrt[3]{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y}$

$2\cdot2\,x^2y\,\sqrt[3]{2y^2}$

$4x^2y\sqrt[3]{2y^2}$

Ex. $\sqrt[4]{-32}$

No Real Solution
due to even root of negative

Ex. $\sqrt[3]{54x^{10}y^{12}}$

$\sqrt[3]{2\cdot3\cdot3\cdot3\cdot x^9\cdot x\cdot y^{12}}$

$\sqrt[3]{2\cdot3\cdot3\cdot3\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y}$ or $\sqrt[3]{27x^9y^{12}}\sqrt[3]{2x}$

$3x^3y^4\sqrt[3]{2x}$

$3x^3y^4\sqrt[3]{2x}$

Ex. $\sqrt[3]{\frac{32}{125}}$

$\frac{\sqrt[3]{32}}{\sqrt[3]{125}} = \frac{\sqrt[3]{2\cdot2\cdot2\cdot2\cdot2}}{\sqrt[3]{5\cdot5\cdot5}}$

$= \frac{2\sqrt[3]{4}}{5}$

What are and how do you solve higher order roots?

Varies

6.6: Higher Order Roots Practice Problems

Simplify:

1. $\sqrt[3]{64}$

$\sqrt[3]{2\cdot2\cdot2\cdot2\cdot2\cdot2}$

4

2. $\sqrt[4]{81}$

$\sqrt[4]{3\cdot3\cdot3\cdot3}$

3

3. $\sqrt[5]{-32}$

$\sqrt[5]{-2\cdot2\cdot2\cdot2\cdot2}$

-2

4. $\sqrt[3]{x^{12}}$

x^4

5. $\sqrt[4]{x^4y^{16}z^{24}}$

xy^4z^6

6. $\sqrt[6]{a^{12}b^{30}}$

a^2b^5

7. $\sqrt[3]{24}$

$\sqrt[3]{2\cdot2\cdot2\cdot3}$

$2\sqrt[3]{3}$

8. $\sqrt[3]{32}$

$\sqrt[3]{2\cdot2\cdot2\cdot2\cdot2}$

$2\sqrt[3]{4}$

9. $\sqrt[4]{80}$

$\sqrt[4]{2\cdot2\cdot2\cdot2\cdot5}$

$2\sqrt[4]{5}$

10. $\sqrt[3]{72x^9y^7}$

$\sqrt[3]{2\cdot2\cdot2\cdot3\cdot3\,x^9y^6y}$

$2x^3y^2\sqrt[3]{9y}$

11. $\sqrt[4]{48x^{11}y^{40}z}$

$\sqrt[4]{2\cdot2\cdot2\cdot2\cdot3\,x^8x^3y^{40}z}$

$2x^2y^{10}\sqrt[4]{3x^3z}$

12. $\sqrt[3]{250x^5y^{30}z^3}$

$\sqrt[3]{5\cdot5\cdot5\cdot2\,x^3x^2y^{30}z^3}$

$5xy^{10}z\sqrt[3]{2x^2}$

13. $\sqrt[3]{\frac{54}{27}}$

$\sqrt[3]{\frac{3\cdot3\cdot3\cdot2}{3\cdot3\cdot3}}$

$\sqrt[3]{2}$

MAT0024C Beginning Algebra Name: _____

Test 4 (Chapter 6) REVIEW

Questions from Chapter 6

1. Simplify: $\sqrt{121}$

11

2. Simplify: $\sqrt{20}$

$\sqrt{4}\sqrt{5}$

$2\sqrt{5}$

3. Simplify: $\sqrt{x^7y^{10}}$

$\sqrt{x^6y^{10}}\ \sqrt{x}$

$x^3y^5\sqrt{x}$

4. Simplify: $\sqrt{63x^2y^9z}$

$\sqrt{9x^2y^8}\ \sqrt{7yz}$

$3xy^4\sqrt{7yz}$

5. Simplify: $5\sqrt{49x^4y^{11}}$

$5\sqrt{49x^4y^{10}}\sqrt{y}$

$5(7x^2y^5)\sqrt{y}$

$35x^2y^5\sqrt{y}$

6. Simplify: $3x^2\sqrt{200xy^{20}z^3}$

$3x^2\sqrt{100y^{20}z^2}\ \sqrt{2xz}$

$3x^2(10y^{10}z)\sqrt{2xz}$

$30x^2y^{10}z\sqrt{2xz}$

7. Simplify: $3\sqrt{5}\cdot\sqrt{8}$

$3\sqrt{40}$

$3\sqrt{4}\sqrt{10}$

$3(2)\sqrt{10}$

$\boxed{6\sqrt{10}}$

8. Simplify: $2\sqrt{3}(5\sqrt{3}-\sqrt{5})$

$10\sqrt{9}-2\sqrt{15}$

$10(3)-2\sqrt{15}$

$\boxed{30-2\sqrt{15}}$

9. Simplify: $4\sqrt{2x}(\sqrt{3x^3}-2\sqrt{4x^4})$

$4\sqrt{6x^4}-8\sqrt{8x^5}$

$4\sqrt{x^4}\sqrt{6}-8\sqrt{4x^4}\sqrt{2x}$

$4x^2\sqrt{6}-8(2x^2)\sqrt{2x}$

$\boxed{4x^2\sqrt{6}-16x^2\sqrt{2x}}$

10. Simplify: $\sqrt{5}(2\sqrt{5}-6)$

$2\sqrt{25}-6\sqrt{5}$

$2(5)-6\sqrt{5}$

$\boxed{10-6\sqrt{5}}$

11. Simplify: $(4\sqrt{3}+6)(2\sqrt{3}-5)$

$8\sqrt{9}-20\sqrt{3}+12\sqrt{3}-30$

$8(3)$ $\quad\vee$

$24-8\sqrt{3}-30$

$\boxed{-6-8\sqrt{3}}$

12. Simplify: $(4\sqrt{3}-2)^2$

$(4\sqrt{3}-2)(4\sqrt{3}-2)$

$16\sqrt{9}-8\sqrt{3}-8\sqrt{3}+4$

$16(3)-16\sqrt{3}+4$

$48-16\sqrt{3}+4$

$\boxed{52-16\sqrt{3}}$

13. Simplify: $\sqrt[4]{81}$

$\sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3}$

3

14. Simplify: $\sqrt[3]{16x^6y^5}$

$\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y}$

$2x^2y\sqrt[3]{2y^2}$

OR

$\sqrt[3]{8x^6y^3}\ \sqrt[3]{2y^2}$

$2x^2y\sqrt[3]{2y^2}$

Review Questions

15. Factor: $30a^{10}b^5 + 20a^5b^6 - 5a^2b^3$

$5a^2b^3(6a^8b^2 + 4a^3b^3 - 1)$

Pull out GCF

16. Factor: $3xy + 3xz - 5y - 5z$

Groups

$3xy + 3xz \quad -5y - 5z$

$3x(y+z) \quad -5(y+z)$

$(y+z)(3x-5)$

✓ $(y+z)(3x+5)$

$3xy + 5y + 3xz + 5z$

$3xy + 3xz - 5y - 5z$

17. Factor: $x^2 + 5x - 24$

$(x+8)(x-3)$

✓ $(x+8)(x-3)$

$x^2 - 3x + 8x - 24$

$x^2 + 5x - 24$

18. Factor: $10x^2 + 21x - 10$

$10x^2 + 21x - 10$ 100 Mult +21 Add +25 -4

$10x^2 + 25x \quad -4x \quad -10$

$10x^2 + 25x \quad -4x - 10$

$5x(2x+5) \quad -2(2x+5)$

$(2x+5)(5x-2)$

✓ $10x^2 - 4x + 25x - 10$

$10x^2 + 21x - 10$

19. Factor: $81x^4 - 49y^6$

$\sqrt{81x^4}$ $\quad$ $\sqrt{49y^6}$

$9x^2$ $\quad$ $7y^3$

$$(9x^2-7y^3)(9x^2+7y^3)$$

✓ $81x^4 + 63x^2y^3 - 63x^2y^3 - 49y^6$

$81x^4 - 49y^6$

20. Solve: $x^2 - x - 42 = 0$

$(x-7)(x+6)=0$

$x-7=0$ $\quad$ $x+6=0$

$+7 \quad +7$ $\quad$ $-6 \quad -6$

$x=7$ $\quad$ $x=-6$

Check

$(7)^2 - 7 - 42 = 0$ $\quad$ $(-6)^2 - (-6) - 42 = 0$

$49 - 7 - 42 = 0$ $\quad$ $36 + 6 - 42 = 0$

$0=0$ $\quad$ $0=0$

21. Solve: $4x^2 + 8x + 3 = 0$

$4x^2+8x+3=0$ $\quad$ +12 Mult +8 Add

$4x^2$ $+2x$ $+6x$ $+3$

$4x^2+2x$ $+6x+3$

$2x(2x+1)$ $+3(2x+1)$

$(2x+1)(2x+3)=0$

$2x+1=0$ $\quad$ $2x+3=0$

$-1 \quad -1$ $\quad$ $-3 \quad -3$

$\frac{2x}{2} = \frac{-1}{2}$ $\quad$ $\frac{2x}{2} = \frac{-3}{2}$

$x=-\frac{1}{2}$ $\quad$ $x=-\frac{3}{2}$

Check

$4(-\frac{1}{2})^2 + 8(-\frac{1}{2}) + 3 = 0$ $\quad$ $4(-\frac{3}{2})^2 + 8(-\frac{3}{2}) + 3 = 0$

$0=0$ $\quad$ $0=0$

22. Simplify: $\frac{2x^2-5x+3}{x^2-1}$

$2x^2-5x+3$ $\quad$ +6 Mult -5 Add

$2x^2 - 2x - 3x + 3$

$2x^2 - 2x \quad -3x + 3$

$2x(x-1) \quad -3(x-1)$

$(x-1)(2x-3)$

$\frac{(x-1)(2x-3)}{(x-1)(x+1)}$

$\frac{2x-3}{x+1}$

23. Simplify: $|4|-|-3|+|2-8|$

$|4| - |-3| + |2-8|$

$4 - 3 + |-6|$

$4 - 3 + 6$

$1 + 6$

$\boxed{7}$

24. Solve for b: $5c = 6a - 2b$

$5c = 6a - 2b$

$-6a \quad -6a$

$5c - 6a = -2b$

$\frac{}{-2} \quad \frac{}{-2}$

$\boxed{b = \frac{5c-6a}{-2} = -\frac{5}{2}c + 3a}$

25. Find the x –intercept for: $-x + 7y = 12$

x	y
-12	0

$-x + 7(0) = 12$

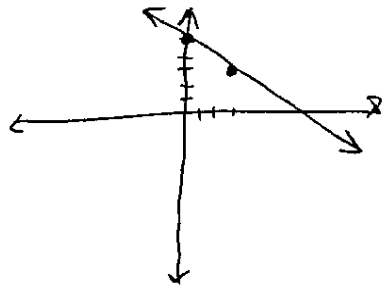
$\frac{-x}{-1} = \frac{12}{-1}$

$x = -12$

$\boxed{(-12, 0)}$

26. Graph: $y = -\frac{2}{3}x + 5$

$m = \frac{-2}{3} \downarrow 2 \rightarrow 3 \quad b = 5$



27. Solve: $10 - 2x \geq 2(2x - 3)$

$10 - 2x \geq 2(2x - 3)$

$10 - 2x \geq 4x - 6$

$-4x \quad -4x$

$10 - 6x \geq -6$

$-10 \quad -10$

$-6x \geq -16$

$\frac{}{-6} \quad \frac{}{-6}$

$x \leq \frac{16}{6} \div 2 \frac{8}{3}$

$\boxed{x \leq \frac{8}{3}}$

28. Simplify: $\frac{(a^2b^3)^4(ab^6)}{a^{10}b^5}$

$\frac{a^8 b^{12} a b^6}{a^{10} b^5}$

$\frac{a^9 b^{18}}{a^{10} b^5}$

$\boxed{\frac{b^{13}}{a}}$

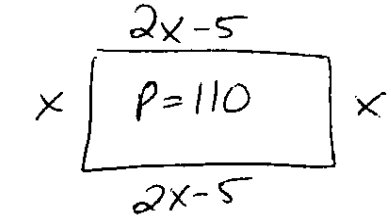
29. Translate into algebraic equation:
The square of a number less than 4 times a number is 5 more than twice a number.

$4x - x^2 = 2x + 5$

30. The length of a rectangular pool is 5 less than twice the width. The perimeter of the pool is 110 feet. Find the length and width of the pool. Label each distance correctly.

Length = $2x-5$

Width = x



$x + 2x - 5 + x + 2x - 5 = 110$

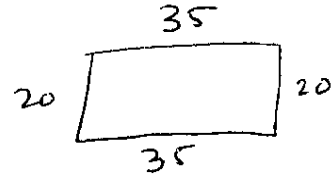
$6x - 10 = 110$

$+10 \quad +10$

$\frac{6x}{6} = \frac{120}{6}$ $\quad x = 20$

Length = $2(20) - 5 = 35$

Width = 20



$20 + 35 + 20 + 35 = 110$

$110 = 110$

31. If a bed cost \$675 after a 20% discount, what was the original cost?

Original − Discount = New Price

$x - .20x = 675$

$.80x = 675$

$\frac{.80}{.80} \quad \frac{}{.80}$

$80\overline{)67500}$

$80\overline{)675000}$

843.75

640

350

320

300

-240

600

560

Original Price

\$843.75